Lösungen - Integrationsmethoden

Aufgabe 1

a. $\int \frac{1}{(x-a)^3} = \frac{-1}{2(x-a)^2} + C$

b. $\int \frac{1}{\sqrt{5-t}}\,dt = -2\sqrt{5-t} + C$

c. $\int \frac{2}{3}(4-3x)^{-5}\,dx = \frac{1}{18(3x-4)^4} + C$

d. $\int \sqrt[3]{4x-1}\,dx = \frac{3(4x-1)^{4/3}}{16} + C$

e. $\int \frac{5}{\sqrt[4]{7t+2}}\,dt = \frac{20(7x+2)^{3/4}}{21} + C$

Aufgabe 2

a. $\int t\cos t\,dt = \cos t + t\cdot\sin t + C$

b. $\int ue^{-u}\,du = (-u-1)\cdot e^{-u} + C$

c. $\int x^2\sin x\,dx = (2-x^2)\cdot\cos x + 2x\cdot\sin x + C$

d. $\int e^y\cos y\,dy = \frac{1}{2}e^x(\cos x + \sin x)$

Aufgabe 3

a. $\int \frac{1}{t^2-1}\,dt = -\frac{1}{2}ln\left|\frac{x+1}{x-1}\right| + C$

b. $\int \frac{13-x}{x^2-x-6}\,dx = -ln\left|\frac{(x+1)^3}{(x-3)^2}\right| + C$

c. $\int \frac{x+2}{x^2+x}\,dx = ln\left|\frac{x^2}{x+1}\right| + C$

d. $\int \frac{y^2-3y+1}{y}\,dy = ln|x| + \frac{x^2}{2} - 3x + C$

e. $\int \frac{u+1}{u-1}\,du = 2\cdot ln|x-1| + x + C$

Aufgabe 4

a. $\int \left(\frac{1}{6t+5} + (6t+5)^3\right)\, dt = \frac{ln|6t+5|}{6} + \frac{(6t+5)^4}{24} + C$

b. $\int \sqrt{a^2 z - a}\, da$ = ……

c. $\int_{-1}^{2} kx^2\, dx = \frac{2}{3}$ = 3k <u>k = 2/9</u>

d. $\int_{1}^{k^2} \frac{1}{z}\, dz = 8$ = $\ln(k^2)$ $k = e^{\pm 4}$

e. $\int_{0}^{k/2} \sin(2x)\, dx = 1$ = $0.5 - 0.5 \cdot \cos(k)$ <u>$k = \pi$</u>

Aufgabe 5

a. $\int_{-1}^{1} y \cdot e^y\, dy$ = $\frac{2}{e}$ = 0.74

b. $\int_{1}^{e} u \cdot \ln(u)\, du$ = $\frac{e^2}{4}$ + ¼ = 2.09

c. $\int_{1}^{2} \frac{4 \cdot \ln(t)}{t}\, dt$ = $2(\ln(2))^2 = 0.96$

d. $\int_{-1}^{3} (2x+1) \cdot e^{(2x+1)}\, dx$ = $\frac{3e^8+1}{e}$ = 3290.27

e. $\int_{0}^{2\pi/\omega} e^{-\alpha t} \cos(\omega t)\, dt$ = $\frac{1}{2t}$

Aufgabe 6

a. $\int_1^2 \left[\int_0^x (3+4t)\,dt \right] dx = \int_1^2 2x^2 + 3x\,dx = \frac{55}{6}$

b. $\int_{-\pi/2}^{\pi/2} \left[\int_0^z \sin x\,dx \right] dz = \int_{-\pi/2}^{\pi/2} 1 - \cos(z)\,dz = \pi - 2$

c. $\int_0^3 \left[\int_2^u (y^2 - y)\,dy \right] du = \int_0^3 \frac{u^3}{3} - \frac{u^2}{2} - \frac{2}{3}\,du = \frac{1}{4}$

d. $\int_0^1 \left[\int_0^x e^{2u}\,du \right] dx = \int_0^1 \frac{e^{2x}}{2} - \frac{1}{2} dx = \frac{e^2-3}{4} = 1.1$

e. $\int_{-1}^1 \left[\int_1^2 (x + y^2)\,dx \right] dy = \int_{-1}^1 y^2 + \frac{3}{2}\,dy = \frac{11}{3}$

Aufgabe 7

a. A = 3, V = 3π
b. A = 2, V = ∞
c. A = ∞, V = π
d. 1
e. –0.5
f. $e^3/2$
g. 1
h. 3π
i. 1